PARASHAT HASHAVUA

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PARASHAT BEMIDBAR

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Rounding of Numbers in the Censes of Bnei Yisrael
By Rav Elchanan Samet

A. THE CENSES IN SEFER BEMIDBAR

Sefer Bemidbar opens with the description of FIVE censes conducted in the second month of the second year after the Exodus from Egypt:

i.	Census of the tribes - chapter 1
ii.	Census of the banners - chapter 2
iii.	Census of Leviim aged one month and upwards - chapter 3
iv.	Census of the firstborn of Israel - chapter 3
v.	Census of the Leviim aged between thirty and fifty - chapter 4

The description of these censes occupies all of parashat Bemidbar and even the first portion of parashat Naso. It extends over four long chapters, numbering a total of 188 verses. The results of the five censes give rise to 41 figures (some of which are stated twice), of which ten are totals of previous figures.

At the end of Sefer Bemidbar, in chapter 26, the Torah describes a census conducted in the plains of Moav in the fortieth year, just prior to the nation's entry into the land. It is on the basis of these two sets of censes - at the beginning of the Sefer and at its conclusion - that Chazal (Mishna Yoma 7:1) refer to Sefer Bemidbar as "the book of counting."

The table below summarizes the data of the various censes in the parashot of Bemidbar and Naso, in comparison with the census in parashat Pinchas. The table shows all 41 figures that appear in chapters 1-4, and will serve as the basis of our discussion.

CENSUS OF THE TRIBES AND CENSUS OF THE BANNERS (BEMIDBAR 1-2) AS COMPARED TO THE CENSUS ON THE PLAINS OF MOAV:

	Bemidbar 1 (Tribes)	Bemidbar 2 (Banners)	Bemidbar 26 (Plains of Moav)
Reuven	46,500		43,730
		151,450	
Shimon	59,300		22,200
Gad	45,650		40,500
Yehuda	74,600		76,500
		186,400	
Yissakhar	54,400		64,300
Zevulun	57,400		60,500
Efraim	40,500		32,500
		108,100	
Menashe	32,200		52,700
Binyamin	35,400		45,600

	Bemidbar 1	Bemidbar 2	Bemidbar 26
Dan	62,700		64,400
		157,600	
Asher	41,500		53,400
Naftali	53,400		45,400
Total	603,550	603,550	601,730

CENSUS OF LEVIIM AGED ONE MONTH AND UPWARDS (BEMIDBAR 3):

Gershon	7,500
Kehat	8,600
Merari	6,200
Total	22,000 (verse 39)
Calculated total	22,300

CENSUS OF FIRSTBORN OF ISRAEL, SELECTION OF LEVIIM INSTEAD, REDEMPTION OF LEFTOVER FIRSTBORN (BEMIDBAR 3):

NUMBER OF FIRSTBORN: 22,273 (verse 43)
NUMBER OF LEVIIM: 22,000

FIRSTBORN REQUIRING REDEMPTION:
22,273 (no. of firstborn) - 22,000 (no. of Leviim) = 273

REDEMPTION OF LEFTOVER FIRSTBORN BY FIVE SHEKALIM EACH:
273 x 5 = 1365 shekalim

CENSUS OF THE HOSTS OF LEVIIM AGED 30-50, COMPARED WITH THEIR NUMBER AGED ONE MONTH AND UPWARDS:

	Bemidbar 4 (aged 30-50)	Bemidbar 3 (1 month +)	Bemidbar 26:62 (1 month +)
Gershon	2,630	7,500	-
Kehat	2,750	8,600	-
Merari	3,200	6,200	-
Total Leviim	8,580	22,000 (as per text)	23,000

B. THREE TYPES OF NUMBERS

A glance at the collection of figures above shows that there are three types:

a. Numbers rounded to hundreds:
These include 11 of the 12 tribes of Israel, as well as the three families of Leviim in their first census (aged 1 month and above) in chapter 3.

b. Numbers rounded to tens:
The tribe of Gad numbers 45,650. In the census of parashat Pinchas we find once again that 11 of the 12 tribes are rounded to hundreds, while the tribe of Reuven is 43,730. Other figures in this category include the families of Gershon (2,630) and Kehat (2,750) in the second census of Leviim (aged 30-50) in chapter 4.

c. Figures ending in units:
All such numbers are related to the census of the firstborn. They number a total of 22,273; the leftover firstborn after the exchange for Leviim is 273, and the number of shekalim by which they are all redeemed is 1,365.

What, then, is the Torah's system for transmitting these numerical data? Does the Torah round to hundreds, to tens, or not at all?

It would seem that the Torah does round numbers, and where an exact figure is given - in the census of the firstborn of Israel - it is because of the need to calculate the number of leftover firstborn that require redemption with the sum of 5 shekalim each.

C. RABBI YESHAYA OF TRANI AND THE ROSH: THE BIBLICAL SYSTEM FOR ROUNDING NUMBERS

As far as I am aware, the earliest commentator who notes that the Torah rounds figures is Rabbi Yeshaya of Trani (author of the famous Tosafot ha-Rid on the Talmud), the greatest of the Italian sages of the thirteenth century. His biblical commentary, called "Nimukim," exists in a few manuscripts in European libraries; it has never been printed (though his commentary on Nakh has been printed, most recently in the Mikraot Gedolot ha-Keter). But it is cited in the Chida's commentary on the Torah, "Pnei David:"

"'The numbers of the tribe of Reuven were forty-six thousand, five hundred' (1:21) - How can this be? Is it possible that there were not one or two less than the figure stated here, or one or two more? We must conclude that the text is not conveying exact figures, as it is written (Vayikra 23:16), 'You shall count fifty days,' where in actuality we count only forty-nine days. Likewise (Devarim 25:3), 'They shall mete out forty lashes' - but they really mete out no more than thirty-nine' (Makkot 22a-b)."

The examples that the Rid chooses to substantiate his claim are somewhat surprising: according to what he says, the Torah usually rounds figures to the nearest ten even when the number is of decisive halakhic significance. Can this be so? His examples are discussed at greater length in Piskei ha-Rosh at the end of Pesachim, concerning Sefirat Ha-Omer:

"And some ask: If the verse says, 'You shall count fifty days,' why do we count only forty-nine days? Their forced explanation for this verse is, 'Until the day after the Shabbat - WHICH ITSELF IS THE FIFTIETH DAY - shall you count,' or alternatively, 'fifty days' - referring to the clause, 'You shall sacrifice…,' such that the meaning is: 'Until the day after the Shabbat you shall count - NOT INCLUSIVE OF THIS DAY - fifty days, and you shall sacrifice a new offering to God.'

But it seems to me that we have no need for these forced explanations. Since the text says explicitly (Devarim 16:9), 'Seven weeks shall you count for yourself,' we do not count more than seven weeks. And the fact that it says, 'You shall count fifty days,' presents no difficulty, FOR THIS IS THE WAY OF THE TORAH: WHEN THE FIGURE IS ONE LESS THAN A TEN, THE TORAH ROUNDS THE NUMBER TO THAT TEN, AND MAKES NO MENTION OF THE SINGLE ONE MISSING. Likewise it says (Bereishit 46:27), 'All the members of Yaakov's household who came to Egypt were seventy,' and also (Devarim 25:3), 'Forty strikes shall they mete….'"

However, the three examples discussed by the Rosh do not necessarily resemble the system of rounding in our parasha. The Rosh speaks of rounding only when a number is within 10% of a round figure. But the numbers of the tribes in our parasha are rounded (except for one, which shall be treated separately) to hundreds, and this is regardless of their fraction of the whole.

This rounding could then refer to anything up to 49 more or less than the number indicated in the text, and this system applies to ALL the figures listed.

D. ROUNDING TO HUNDREDS OR TO TENS - ARTICLE BY A. MERZBACH

Thus far our assumption has been that in recording the censes, the Torah rounds figures to hundreds, as it would seem from the great majority of those that appear in chapters 1-4. But in section B. above, we note that there are a few figures in these chapters (and another one in parashat Pinchas) that end in tens, and not in hundreds. They are:

a. The tribe of Gad in our parasha   45,650 (1:25)
b. The tribe of Reuven in par. Pinchas   43,730 (26:7)
c. The family of Kehat aged 30-50   2,750 (4:36)
d. The family of Gershon aged 30-50   2,6(4:4)

What is the reason for these exceptions to the system of rounding to hundreds?

Prof. Ely Merzbach, of the department of mathematics at Bar-Ilan University, addresses this question in his article, "The Censes of Bnei Yisrael in the Desert" (published in the journal "Higgayon" - Studies in Rabbinical Thought, vol. 5, 5761). Here are some excerpts:

"It is always possible to attribute this phenomenon (of exactly rounded numbers) to a miracle, or coincidence, without any explanation (as some commentators have attempted to do). But explanations of this sort are rejected by the major commentators with the simple claim that a miracle must have some significance, or some benefit.

It seems to me that it is possible… [to explain the phenomenon] based on the following principles, which refer to fairly large numbers (and certainly to numbers greater than 5,000).
.   When the figure obtained is in whole tens (without units), the Torah records it as is, without rounding it.
.   When the figure obtained is not in whole tens, then the Torah rounds it to the nearest hundred.

The logic behind this system is simple: if a number ending in units already requires rounding, it is rounded to hundreds (with some small margin of inaccuracy). But if the figure ends in tens, it is left as is.

If we examine the data in the Torah, this becomes completely clear. In each of the two censes of Bnei Yisrael in the desert, 11 out of 12 figures are multiples of hundreds, while one (the tribe of Gad in the first census, and Reuven in the second one) is a multiple of tens. The probability of any number ending in zero but not being a multiple of 100 is 9/100. Therefore if any 12 numbers are chosen, we can expect the incidence of appearance of numbers with this characteristic to be 12 x 9/100 = 1.08. In other words: on average, out of 12 numbers, one will be a multiple of tens (and not a multiple of hundreds).

Moreover ... the greatest probability exists, once again, when there is exactly one number of this sort out of 12 numbers… Concerning the censes of the Levite families we could obtain similar results, but when the number of data is small (there are only three families), no statistical test may be applied."

E. ALTERNATIVE EXPLANATION FOR NUMBERS ENDING IN TENS IN CHAPTERS 1-4

Merzbach's two rules for rounding figures explain all four exceptional figures listed at the beginning of section D. above, and even match the statistical probability of the phenomenon of the two numbers - the tribe of Gad and the tribe of Reuven - in the two censes held in the desert. But the "simple logic" that he employs is actually not so simple. He writes, "If a number ending in units already requires rounding, it is rounded to hundreds. But if the figure ends in tens, it is left as is." We may ask: if a number ending in tens is considered a round number, then why are numbers ending in units not rounded to the nearest ten, thereby diminishing the maximal inaccuracy from 49 to 4? The proposition that "If a number… already requires rounding, it is rounded to hundreds" is not a mathematical one; it is a matter of personal taste.

Setting aside for a moment the exception of the tribe of Reuven in parashat Pinchas, focusing instead only on the numbers in chapters 1-4, we may solve the difficulty of the three exceptional figures without reliance on Merzbach's two rules.

Let us first address the censes of the families of Kehat and Gershon. These are small numbers - between 2,000 and 3,000, making it unreasonable to round to hundreds. It would appear, then, that the Torah here rounded to tens, and therefore two of the three figures end in tens. Apparently, the number of the family of Merari (3,200) is also rounded to the nearest ten - which happens to also be a whole hundred.

Let us illustrate our claim that the system of rounding figures in the Torah varies in accordance with the size of the numbers (and perhaps even with the importance of the items being counted). In chapter 31 of Sefer Bemidbar (verses 32-35), we find the various types of booty captured in the war against Midian listed in descending numerical order:

a. flocks   675,000
b. cattle   72,000
c. donkeys   61,000
d. prisoners 32,000

All of these figures end in thousands, and it appears that they are rounded to the nearest thousand. This system is applied because of the large numbers involved.

In Sefer Shemuel we read the figures for two national censes conducted prior to a war, and an additional, general population census. The data are as follows:

i.   Shemuel I 11:8 - The census at Bezek preceding the war with Nachash the Amonite:
     Bnei Yisrael: 300,000
     Men of Yehuda: 30,000
ii. Shemuel I 15:4 - The census at Telaim preceding the war against Amalek:
     The nation: 200,000
     Men of Yehuda: 10,000

Here, again, the figures seem to be rounded to thousands, and the figures of Bnei Yisrael may even be rounded to tens of thousands. This possibility is strengthened by the third census, where David numbers the entire population:
iii. Shemuel II 24:9 -
     Yisrael: 800,000
     Yehuda: 500,000

Now we must deal with the number of the tribe of Gad: 45,650. Why is this number not rounded to the nearest hundred? Perhaps because it ends precisely with 50, and therefore cannot be rounded either upwards or downwards.

But we may suggest a slightly different idea: since the exact number 50 cannot be rounded, it may itself be considered a rounded number, in a sense, even within a system of rounding to hundreds. Therefore it is possible that where the real number is close to fifty, the number is rounded to 50 rather than to 100 (thereby diminishing the inaccuracy that would result from rounding to 100).

F. THE EXCEPTION OF THE NUMBER OF REUVEN IN THE CENSUS CONDUCTED IN THE PLAINS OF MOAV

We must now return to the question that still remains open - the number of the tribe of Reuven in the census of parashat Pinchas: 43,730. The principles we have employed thus far in resolving exceptional figures in chapters 1-4 are of no benefit here.

The purpose of the census conducted in the plains of Moav is stated explicitly at its conclusion:

(26:53-54) "To these you shall divide the land as inheritance, by the number of names… to each according to his number shall his inheritance be given."

This is a census of those who will inherit the land, and it is conducted a short time prior to entering the land. For this reason the Torah lists the households comprising each tribe, since the tribal inheritance will be divided among those households (see Ramban on 26:5).

Each section summarizing the details of the census of a certain tribe is structured in the same way. For example:

(48) "The children of Naftali by their families:
Of Yachtze'el - the family of the Yahtze'eli;
Of Guni - the family of the Guni;
(49) Of Yetzer - the family of the Yitzri;
Of Shilem - the family of the Shilemi.
(50) These are the families of Naftali by their families and by their number: forty-five thousand four hundred."

There are two clear exceptions to this fixed pattern: the tribe of Reuven and the tribe of Menashe. Following the list of the census of Reuven (which is no different from that of the other tribes), and the total, we find the following appendix:

(8-11) "And the children of Palu [mentioned previously as the patriarch of one of the families of Reuven] - Eliav.
And the children of Eliav were Nemuel and Datan and Aviram - the same Datan and Aviram ... who strove against Moshe and against Aharon ...
And the earth opened its mouth, and it swallowed them and Korach when the congregation died, when the fire consumed the two hundred and fifty men, and they became a sign.
But the sons of Korach did not die."

The question is clear: what is this reminder of the sin of Datan and Aviram and their punishment doing in the middle of the data gleaned from a census of the generation that is about to enter the land?

The second exception to the pattern concerns the tribe of Menashe. One of the households of this tribe is that of the Chefri (verse 32), and we find the following insert:

(33) "And Tzelafchad, son of Chefer, had no sons, but daughters.

And thnamof thdaughters of Tzelafchad were Machla and Noa, chogla, Milka and Tirtza."

And then the census of the tribe of Menashe concludes:

(34) "These are the families of Menashe by their number; fifty-two thousand seven hundred."

Again, what does this parenthetical information concerning the daughters of Tzelafchad have to do with the census? It is clear that these daughters are not included in the census of "all the congregation of Bnei Yisrael… all those who went out as the army of Israel."

This question is immediately resolved at the end of the census data, when the Torah recounts the story of the daughters of Tzelafchad. There it becomes clear that there is a connection between our two questions:

(27:1) "And the daughters of Tzelafchad, son of Chefer, drew close… [who were] of the family of Menashe, son of Yosef - and these are the names of his daughters…
(2) And they stood before Moshe and before Elazar… saying:
(3) Our father died in the desert, BUT HE WAS NOT AMONG THE CONGREGATION THAT REBELLED AGAINST GOD, THE CONGREGATION OF KORACH; rather, he died for his own sin, and he had no sons.
(4) Why should the name of our father be disadvantaged in relation to his family because he had no son? Give us a possession among our father's brothers."

Moshe brings their claim before God, and receives the answer:

(7) "The daughters of Tzelafchad speak rightly; they shall surely be given a possession for inheritance among the brothers of their father…."

The timing for the plea by the daughters of Tzelafchad is at the end of the census that is meant to pave the way for a division of the land among all the MEN counted: "When the daughters of Tzelafchad heard that the land was being divided among the tribes to males and not to females, they gathered together…" (Sifri, Pinchas).

The connection between the census and the claim by these women is hinted at in the mention made of their names in the list of the families of Menashe. The Torah hints to us that although they were women and were therefore not included in the census, they would nevertheless be permitted, by God's command, to be included in the PURPOSE of the census - i.e., in the inheritance of the land by merit of their father, who took part in the Exodus (and it was those who left Egypt who would inherit the land). Thus, the total number of the tribe of Menashe, as stated in verse 34, after the names of the daughters of Tzelafchad in verse 33, includes them.

In justifying their demand to receive the possession that their father should have been given in the land, Tzelafchad's daughters emphasize his right:

(3) "Our father died in the desert, but he was not among the congregation that rebelled against God, the congregation of Korach…."

It would seem, from their words, that the congregation of Korach - Datan, Aviram and all their families - lost their right to an inheritance in the land. Indeed, Chazal deduce (Bava Batra 117b) from their words here that "those that complained, and the congregation of Korach, had no portion in the land."

Now we understand why the sin and punishment of Datan and Aviram are mentioned after the census of the tribe of Reuven. As the Ramban explains (v. 9):

"The text mentions this to tell us… that Datan and Aviram and all that they had were swallowed up… as our Sages taught, to hint to us that they lost their portion in the land, even though they were among those who left Egypt and were worthy of inheriting."

The mention of Datan and Aviram, and of the daughters of Tzelafchad, in the midst of the census data therefore have two opposite purposes. Neither, admittedly, is actually included in the census: Datan and Aviram and their families were no longer alive when it was conducted, while Tzelafchad's daughters were not counted since they were women. But the Torah mentions Datan and Aviram in order TO REMOVE THEM FROM THOSE INCLUDED IN THE PURPOSE OF THE CENSUS - the inheritance of the land, while the women are mentioned in order to include them in this inheritance. Were it not for the sin of Datan and Aviram, they would be included in the census, and would merit to receive their proper portion of the land. For this reason they are mentioned AFTER the total number of the tribe of Reuven - in order to illustrate thereby their absence from this total. The daughters of Tzelafchad, in contrast, are mentioned PRIOR to the summary of the number of the tribe of Menashe, and this is meant to INCLUDE them in the census of their tribe as people who were going to inherit the land.

We may now venture a guess at the reason for the non-rounded number (to hundreds) of the tribe of Reuven - 43,730: This number hints to the lack, the missing members of this tribe - those who should have inherited but lost this right. Several people should have been included in this census but were absent because they were swallowed up by the earth. Had the number been rounded (to 43,700), as in the case of the other tribes, we would not have the sense that the tribe of Reuven was missing some of its numbers. Since the figure ends in 30, we may assume that the number of men aged twenty and upwards who were swallowed by the earth was several dozen.

The appendix to the census of the tribe of Reuven - the story of the sin of Datan and Aviram and their sin - is simply the explanation for the special number, and the lack, in that tribe. The figure is deliberately not rounded in order to hint at a special problem that affected only this tribe.

When Moshe blesses the tribes of Israel before his death, he begins his blessing of Reuven as follows:

(Devarim 33:6) "May Reuven live and not die, and may his people be numerous."

Targum Onkelos translates this with an added dimension:

"May Reuven live [in the world of] eternal life, and not die a second death, and may his people receive their inheritance by their number."

(Translated by Kaeren Fish.
The unabridged Hebrew version of this shiur is archived at:
http://www.vbm-torah.org/hparsha-7/hparsha7.htm.)